Subitizing: Vision Therapy for Math Deficits

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ABSTRACT

Subitizing is the immediate visual perceptual apprehension and enumeration of a small set of elements. Subitizing deficits are correlated with difficulty in math at all ages. The incidence of individuals with mathematics learning disability (MLD) is between 6-7% of the population. This is unfortunate because math skills are of prime importance in everyday life enabling us to understand number concepts and do calculations. Math ability is essential for many occupations and professions. Subitizing is a basic skill that young children and many animals exhibit. Subitizing has been shown to be a precursor of math skills. A subitizing computer program has been designed and based upon theories and experimental data appropriate for improving math skills. It consists of a diagnostic test and four therapy programs: Flash-Number, Comparison-Spatial, Visual Counting, and Temporal Visual Counting. Subitizing therapy appears to improve both subitizing and math abilities.

Keywords: subitizing, math, dyscalculia learning disability

For the things of the world cannot be made known without knowledge of mathematics.
Roger Bacon

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What is Subitizing?

Three pictures hang in front of a six month child. The first shows two dots, the others show 1 dot and three dots. The infant hears three drumbeats. Her eyes move to the picture with three dots. Young children, even infants,[1] spontaneously have the ability to recognize and discriminate small numbers of objects.[2] Some children can and some cannot immediately name the number of pips showing on dice or the number of dots on a domino. This ability to instantly see how many is called subitizing from a Latin word meaning suddenly. Subitizing is the direct visual perceptual apprehension of the numerosity of a group. It is an accurate quantitative evaluation of small sets without explicit (either internal or external) counting.[3] It is an ancient and intuitive sense that we are born with and that we share with many animals[4] including pigeons, monkeys, elephants and rats.

When we visually enumerate objects, two distinct patterns of performance emerge, subitizing and counting. Subitizing is utilized for approximately four items or fewer. Performance is fast and accurate with relatively little increase in response times as the number of items increases. Counting is utilized for more than four items and performance is slower and less accurate and response times increase steeply as the number of items increase.[5] Subitizing has been taken to reflect the operation of one or more specialized mechanisms or processes that are able to enumerate small numbers of items in a spatially parallel and rapid manner. When this mechanism cannot be applied, enumeration proceeds via a slower more serial process.[6] There are two distinct types of counting or enumeration: Spatial and Temporal.[7]

Spatial Enumeration is the counting of units that are all present at one time in different spatial locations. Determining the number of dots in a visual display is spatial enumeration and requires visual spatial working memory. The converse of spatial enumeration is temporal enumeration. Determining the number of times a light flashes in the center of a visual display involved temporal enumeration, and requires phonological working memory. It has been suggested[8,9] that eye movements may be necessary for accurate enumeration beyond the subitizing range of four items. Watson, Maylor and Bruce[10] recently investigated the role of saccades in subitzing and counting. They examined the role of eye movements in enumeration. The main findings showed that when eye movements were allowed, saccadic frequency increased much more with numerosity in the counting range than in the subitizing range, and when eye movements were prevented there was a selective and negative effect on the enumeration rates in the counting range and no effect on enumeration rates in the subitizing range.

History

Jevons,[11] probably the first scientist to report on subitizing, observed that enumerating small numbers seemed to happen "all at once" with very little error. In contrast, enumerating larger numbers was laborious, sequential, and error prone. Most adults seemed to scan the display area by area, enumerating the items within an area, adding this sum into a running total, and then moving to the next items. This process has been termed *group and add* by Klahr and Wallace.[12] They also suggest that different processes are required for subitzing and counting because of limitations in short term visual memory. Others contend that the two operations reflect two different levels along a continuum of complexity.[13] Over eighty years ago Douglas[14] suggested that subitizing was a developmental prerequisite to counting and number concept. Even earlier, Freeman[15] concluded that since measurements focused on the whole and counting focused on the unit, only subitizing focused on both the whole and the unit, subitizing is basic to number ideas. Significant research over the years has shown that individuals who have poor subitizing skills have

1. poor number sense
2. poor basic arithmetic skills
3. poor in higher mathematical concepts

Subitizing is particularly difficult for some special populations including mild and moderate mental retardation[16], cerebral palsy[17], those with Alzheimer's disease[18], and Turner syndrome.[19] Recent research has shown that subitizing responds to therapy.

Importance of Arithmetic

Arithmetic is of prime importance in everyday life, enabling us to comprehend number concepts and perform calculations. Budgeting our time and monetary resources, reading calendars, balancing a checkbook, locating an address, following a recipe are examples of our dependence on elementary arithmetic skills. Advanced mathematics is founded on our basic math skills. Most significantly, sound math ability is a requisite for numerous occupations and professions. According to Paulos,[20] the consequences of mathematical illiteracy are very widespread and may include misinformed governmental policies and confused personal decisions. During the school years, poor achievement in mathematics stigmatizes a child and, like a reading disability, contributes to feelings of low self-esteem. Poor skill in basic mathematics has been shown to have a greater negative effect on employment opportunities and job retention than poor literacy skills.[21] Despite the recognized importance of math deficits U.S education suffers from shortcomings that put even children possessing adequate intellectual abilities are at risk for low mathematics achievement. Whatever the reason for this, the topic of learning disabilities in mathematics requires serious attention.[22]

Mathematics Learning Disability (MLD)

MLD or dyscalculia is defined as a specific learning disability affecting the normal acquisition of arithmetic skills.[23] Dyscalculia refers to a wide range of learning difficulties involving math difficulties. There is no single form of MLD – difficulties can vary from person to person and can change throughout a lifetime. The National Center for Learning Disabilities[24] in 2006 compiled a list of warning signs by age:

Young Children

- Difficulty learning to count
- Trouble recognizing printed numbers
- Difficulty tying together the idea of a number (4) and how it exists in the world. (4 horses, 4 cars, 4 children)
- Poor memory for numbers
- Trouble organizing things in a logical way – putting round objects in one place and square ones in another

School-Age Children

- Trouble learning math facts (addition, subtraction, multiplication, division)

- Difficulty developing math problem-solving skills
- Poor long term memory for math functions
- Not familiar with math vocabulary
- Difficulty measuring things
- Avoiding games that require strategy

Teenagers and Adults

- Difficulty estimating costs like grocery bills
- Difficulty learning math concepts beyond the basic math facts
- Poor ability to budget or balance a checkbook
- Trouble with concepts of time, such as sticking to a schedule or approximating time
- Trouble with mental math
- Difficulty finding different approaches to a problem

Genetic, neurobiologic, and epidemiologic evidence indicates that MLD is a brain-based disorder. The etiology of MLD is multifactorial, including genetic disposition, environmental deprivation, poor teaching, mathematical anxiety, and neurologic deficits. The prevalence of MLD ranges between 6-7% in a number of studies in both the U.S, and other countries.[25] It appears that many – perhaps more than 50% of children with MLD also have dyslexia and other reading disabilities, and that many children with reading disability also have difficulties learning basic arithmetic.[26] Pennington[27] believes that we must differentiate between math problems found in dyslexics and nondyslexics. The dyslexics have problems memorizing math facts (e.g. multiplication tables); doing multistep calculations; and, of course, understanding word problems. The nondyslexics, on the other hand, have difficulty in conceptualizing mathematical concepts. It is sometimes premorbid with a variety of other deficits including Attention Deficit Hyperactivity Disorder.

Is Subitizing a Basic Skill?

Subitizing is instantly recognizing a number of items without using other mathematical processing. Children can subitze directly through interaction with the environment. Very young children can subitize one or two sets but cannot count them. Subitizing emerges before counting and is a necessary precursor of the basic skill of counting. Children, for example, might "recognize 3 items" without using any learned math knowledge. In Butterworth's[28] view of the development of math abilities, the core component is the ability to "categorize the world in terms of numerosity" – the ability to recognize, represent, and manipulate cardinal values... He also stated,[29] "Contrary to what Piaget and others have proposed, infants seem to respond to the numerical properties of their visual world, without benefit of language, abstract reasoning, or much opportunity to manipulate tier world." Butterworth (1999) posits that subitizing is one of the basic component skills that underlie the development of numeration and calculation.

Subitizing: A Precursor to Math Skills?

Penner-Wilger[30] and her associates were the first to do a research study on the relationship between subitizing and early math skills including counting, addition, and performance on a standardized test of math achievement.

The participants were 146 Grade 1 children of relatively high socio-economic status were selected from an ongoing longitudinal study. The children (71 girls and 75 boys) ranged in age from 5-7 years old (M = 82 months). All of the computer tasks were completed by most children in a half hour session.

All of the computer tasks were presented using software developed specifically for the project. The children initiated the trials themselves by pressing the spacebar. Response times (RT) were measured from the point at which the stimuli appeared, until the experimenter pressed the stop-time key when the child spoke their response.

The subitizing materials displayed on the computer screen consisted of a set of 1-6 circular red target objects. The subjects were instructed to respond with the number of objects, out loud, as quickly as possible. To promote accuracy the targets remained on display until the child's response was entered by the experimenter. There were 18 trials, preceded by two practice trials of two and seven objects. Half of the trials were within the subitizing range (1-3), and half in the counting range (4-6).

The measure of interest was the RT slope as a function of set size. To compensate for the variability in RT of this small number of trials, median response times were calculated for each set size for each child. The best fitting regression line through these medians was calculated for each child. The slope values were used as the dependent measure.

The math skills utilized included:

- Digit Recognition/Next Number: For Digit Recognition the experimenter asked the child,"What number is that." For Next Number task the child was asked to respond with the number," that comes next when counting." The total number of correct responses was used as the dependent measure.

- Numeration: Concepts such as quantity, order, and place value were measured with the Numeration subtest of a multi-domain diagnostic test, the Key Math Test Revised, Form B.
- Calculation Skill: Mathematical skill was assessed with Calculation subtest from the Woodcock-Johnson Psycho-Educational battery – Revised (Woodcock & Johnson 1989). This subtest begins with small mathematical problems in both horizontal and vertical formats. The problems progress in difficulty and include addition, subtraction, and multiplication.

Results of the testing showed that subitizing was correlated with both number systems knowledge and calculation skill. Children able to enumerate 1-3 items without counting performed better in mathematics. Subitizing predicted calculation skill both directly and indirectly through number system knowledge. Subitizing exerted influence on calculation skill beyond that accounted for by number system knowledge. They found that subitizing ability predicted counting speed; counting speed in turn predicted addition speed and addition accuracy. This is consistent with the concept that subitizing and counting are separate but overlapping processes and that counting depends on subitzing.

Benoit & Associates[31] in a well designed experiment compared counting with subtizing in number word (e.g. one, four, ten) acquisition. Subitizing appears to be the developmental pathway for acquiring the meaning of the first few number words since it allows the child to simultaneously grasp the whole and the elements at the same time. It is a necessary component for the development of numerosity. When the task becomes greater than the subitzing ability of an individual the slower process of counting is utilized. This is confirmed by Landerl, Bevan and Butterworth[32] who found dyscalculic children were slower at number comparison compared to a control group and that they exhibited deficits in subitizing. They were poor in the *group and add* procedure of Klahr and Wallace described previously.

Halberda, Mazzocco, and Feigenson[33] claim that many basic numerical intuitions are supported by an evolutionarily ancient approximate number system (ANS) that is shared by adults, infants, and animals – these groups can all represent the number of items in visual arrays without verbally counting. This is a subitizing task. In their research study conducted at John Hopkins University 64 14- year- olds were tested at length on the discriminating power of their ANS. The teenagers sat at a computer as a series of slides with varying numbers of yellow and blue dots flashed on a screen for 200 milliseconds. Given the antiquity and ubiquity of the nonverbal number sense, the researchers were impressed by how wildly it varied. There were many with fine powers of discrimination, able to distinguish ratios of 9 blue dots for every 10 yellow ones. Others performed at a level comparable to a 9 month old baby. The researchers found a robust correlation between ANS (subitizing) at age 14 and results on a series of standardized math tests. Dr. Halberda said," We discovered that a child's ability to quickly estimate how many things are in a group significantly correlates with that child's performance for every single year, reaching all the way back to kindergarten.

Fischer et al[34] tested the hypothesis that children with difficulties in acquiring basic arithmetic skills exhibit developmental deficits in subitizing. The experimental group consisting of children 7-17 years old with MLD was slower with poorer accuracy even for subitizing targets with item numbers below 3 or 4. They found that in the MLD subjects, developmental deficits in the specific visual capacity of subitizing and counting were estimated to be between 40% and 78% (increasing with age). The control group did not exhibit these deficits. They concluded that the deficits in the basic visual capacity of subitizing contributes to the problems encountered by children with anomalies in acquiring basic arithmetic skill. They also report that preliminary data from their laboratory shows that up to 60% of dyslexic children also suffer from deficits in subitizing and MLD children may also suffer from deficits in saccade control. Subitizing is a precursor to math skills.

Does Subitizing Therapy Improve Subitizing and Arithmetic Skills?

Wilson et al,[35] a French group of cognitive scientists, describe the design of software that trains children on a numerical comparison task. The overall design of the software displays two screens which contain different quantities of items ranging from 1-9. The children choose which screen has the

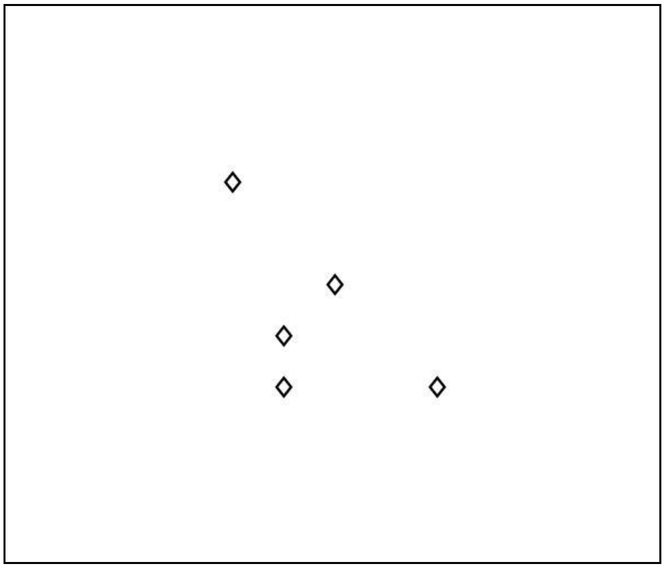

Figure 1: *FLASH – The Flash procedure presents the patient with a group of items that are flashed very quickly on the screen. The patient is required to determine how many there are, not what they are.*

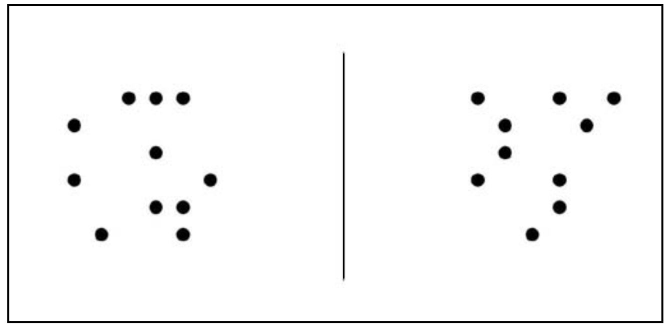

Figure 2: *NUMBER COMPARISON – The screen shows two side by side arrays containing a number of geometric figures. One of the arrays has a larger number of items than the other. The patient is required to identify the larger array as quickly as possible.*

larger number. The task involved subitizing for small quantities of items and a combination of subitizing and counting for larger quantities of items. The program is based on a concept of the cerebral representation of number sense and the hypothesis that dyscalculia is due to a core deficit in number sense or in the link between number sense and symbolic number representation. A companion paper[36] suggests that the software may be useful for the remediation of dyscalculia. A study was carried out as the first step in an ongoing series of tests of the efficacy of the software.

Twenty-two children aged 7-10 years were recruited from three schools in Paris by teacher recommendation, which was based on the observation of persistent or severe difficulties in mathematics. After exclusion screening, 13 children were selected for the study.

Children were tested using a WISC-III short form consisting of vocabulary, picture completion, and arithmetic. Three children were excluded because they had an estimated IQ of less than 80 using the non-arithmetic tests and 2 children were excluded because they did not have a below average score on the arithmetic test. Three children were excluded for other reasons. Four of the children who participated in the study were eventually excluded because of extended absences or disruptive behavior.

The final sample for the study was nine children between the ages of 7 and nine (M=8.1 years). The arithmetic subtest scores of the WISC for the final sample ranged from 1st to 37th percentile with an average score of 12th percentile.

The study took place at school during school hours over a period of 10 weeks. Screening and pre-testing occurred in the first two weeks, children were on vacation during the next two weeks, and then completed the therapy in the fifth to ninth weeks. This consisted of a supervised one half-hour session using the software for each child four days a week for a maximum of ten hours. Due to absences, the average was eight hours. During the tenth week the children were post-tested. The test battery was primarily computerized and tasks included were enumeration in both the subitizing and counting range, numerical comparison, addition, and subtraction.

The results of this limited clinical research reported that children showed significant (p=0.003) improvement reaction times and accuracy of subitizing between the pre- and post-test but no difference in the counting range. Number comparison accuracy and speed showed a significant (p=001) Addition accuracy did not show a significant increase but subtraction accuracy increased by an average of 23% (p=0.04). These results are unlikely to be caused by general motivation, placebo, or attention effects because they were specific to particular tasks and to particular conditions within tasks. They concluded that the children's results of pre- and post-testing on several tasks suggested that the training was successful in improving basic numerical cognition. This study was limited by the small number of participants and the fact that there was no control group. Nevertheless, the results from the enumeration task supported the core deficit hypothesis, with the speed of enumeration increasing by 300 msec for numerosities in the subitizing range of 1-3, while showing no change in the counting range of 4-8. This is consistent with findings

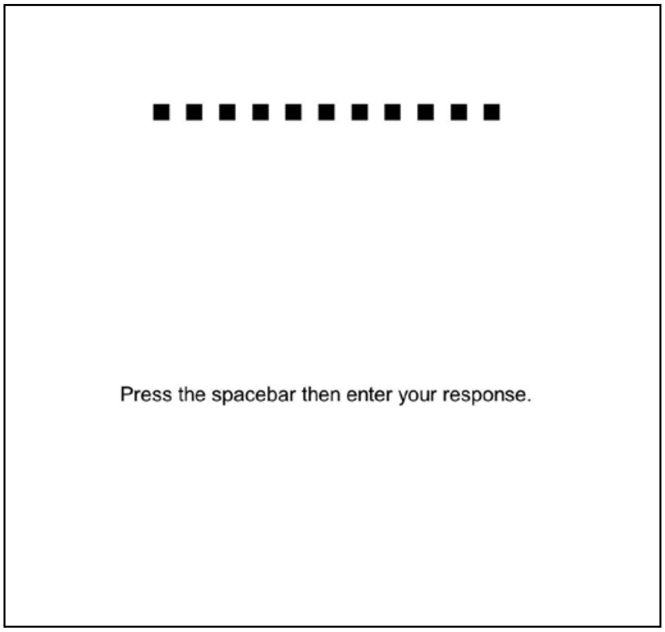

Figure 3: *SPATIAL VISUAL COUNTING – This task presents an array of geometric figures that the patient is required to enumerate as quickly as possible. The array remains on the screen until the patient answers or the designated time elapses.*

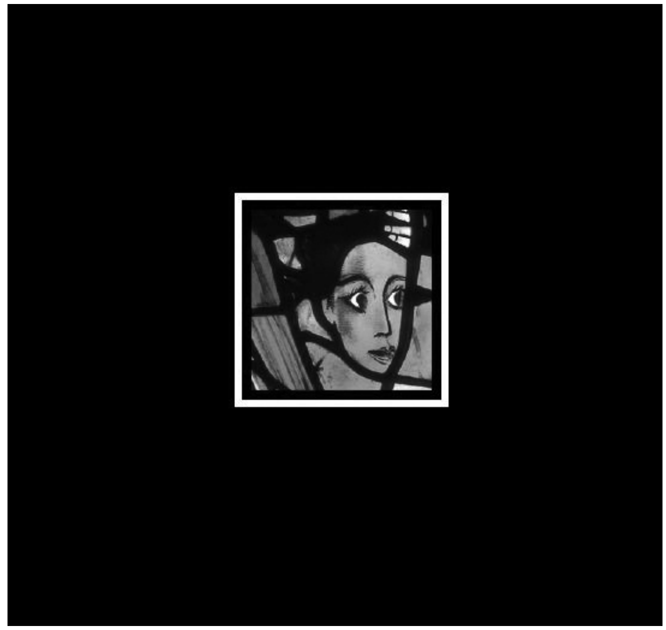

Figure 4: *TEMPORAL VISUAL COUNTING – Temporal counting requires enumerating a number of individually presented items in a speeded situation. A target picture is displayed. Then a number of the specified items are displayed consecutively in the same location for a short time interval. The patient is required to count the total number of pictures displayed. At higher levels, a distractor will be presented on the screen before the patient can record a response.*

that dyscalculic children show impaired subitizing[37] and with the association between subitizing and number sense deficits seen in adult patients.[38]

Fischer and his associates completed a second study[39] in which visual stimuli (small circles) were flashed on a computer at fast exposures. Subjects were selected on the basis of their poor performance in basic arithmetic skills. The test group contained 74 subjects in an age range of 7 to 14 years. Daily training sessions of 10 to 20 minutes were done at home for 3 weeks. The number of children who reached the range of the normal control children in both variables was estimated at 85%. Including the number of children who improved their pre-training value by a certain percentage the success rate would be estimated at 95.9%.

They also analyzed the effect of the therapy on arithmetic skills. A standardized math test (DEMAT) was administered pre- and post-therapy. The test group gained 3.2 points while the control group lost 0.5 points. This difference was significant with p=0.016. A second group showed similar gains. Three domains of the DEMAT were not improved by the test group. The first of them deals with the characteristics of numbers, the second with division, and the third with money. These three are only loosely related to subitizing and number counting. When these variables were removed from the analysis the differences between the test group and the control group became highly significant (p=0.001)

The authors feel there was not a significant placebo effect because not all of the DEMAT variables were improved by the training. The domains not improved were (1) the characteristics of numbers, (2) division, and (3) money. These three are only loosely related to subitizing and number counting. Furthemore, the visual capacities of subitizing and number counting are low level basic visual functions, probably not under control of psychological factors. They conclude, "This study confirmed the idea of Dehaene,[40] who discussed that one reason for the problems in dyscalculia, is the poor development of the sense of number on the basis of subitizing." Subitizing therapy improves subitizing. Subitizing therapy improves math skills. Subitizing therapy is vision therapy for math deficits.

A Subitizing Computer Program

Theoretical Basis

The subitizing computer program is a new home VT program whose design is based on the theories and experimental studies reviewed in this paper. Four related tasks of subitizing and visual counting are utilized. This not only provides variety but because it also trains various aspects of subitizing.

Diagnostic Procedures

1. The patient's presenting history is usually sufficient to indicate that a math deficiency is present.
2. The National Center for Learning Disabilities list of warning signs for mathematics learning disability (MLD) is a valuable tool.
3. Special attention should be paid to patients with learning disabilities, dyslexia, and attention deficit hyperactivity disorder because of the high co-morbidity with MLD.
4. The Subitizing Test evaluates the patient's subitizing ability level and should be administered to all patients.

Optometric Vision Therapy as a Learning Process

It is generally agreed that optometric vision therapy is a learning process that is subject to the laws of learning that have been developed as far back as the early Greeks. Learning may be described as a "relatively permanent change in behavior or in behavioral potentiality that results from experience and cannot be attributed to temporary body states induced by illness, fatigue, or drugs".[41] There are many theories of learning that can be utilized in vision therapy. The technique of operant conditioning[42] has proven very valuable and computers lend themselves to its use. The advantages of computers included in the subitizing program are:

- Patient Acceptance – most patients enjoy working with computers and are motivated by them. It allows us to capitalize on the fascination children have for computer games which makes it easier to provide intensive therapy which might otherwise become boring for them.
- Flexibility – computers are adaptable to many conditions and can adjust to various demands of the doctor and patient.
- Sound Principles of Learning – the operant condition technique of starting learning at a low level so that the patient will succeed, and then increasing difficulty levels in small increments is well suited to computers. This maintains the difficulty of a task, while minimizing failure thus providing an ideal level of the cognitive stimulation needed for progress.
- Programs Are User Friendly – patients are not overwhelmed by difficult levels or by too much material and they have a high success rate.
- Large Number of Stimuli – Patients do not get tired of the same material used over and over again and enjoy the versatility of the techniques.
- Overt Responses required – Patient must make some motor or verbal response or both which enhances the learning process.
- Feedback to Patient – Patient knows immediately if response is correct or incorrect

The tasks included in the program are:

Flash

This is the basic subitizing program. Items, consisting of geometric figures, are presented to the patient at speeds that do not allow saccadic eye movements. The patient is required to answer the question, "How many did you see?" This differs from a tachistoscopic task which requires the patient to answer the question, "What did you see?" The patient is urged to respond as quickly as possible since response time is an important factor.

Perceptual Analysis of Flash

Subitizing – The ability to instantly know the number of items in a small set of elements without using any mathematical processes.

Simultaneous Processing – The ability to integrate separate elements into a whole. Its essential characteristic is that information is handled in a holistic fashion with a gestalt-like integration of information.[43]

Visual Perceptual Speed – The ability to complete a low level cognitive task rapidly and accurately. Speed of processing is basic to all perceptual and cognitive learning.

Visual Reaction Time – The ability to react quickly to a visual stimulus.

Visual Attention – the ability to focus or act upon relevant visual stimuli and to ignore irrelevant visual stimuli.

Number Comparison

In this task the screen shows two side by side arrays containing items of identical geometric figures. One of the arrays has a larger number of items than the other. The patient is required to identify the larger array as quickly as possible since response speed is an important factor. The arrays remain visible until the patient answers or designated time elapses.

Perceptual Analysis of Number Comparison

Subitizing – The ability to instantly know the number of items in a small set of elements.

Simultaneous Processing – the ability to integrate separate elements into a whole.

Visual Perceptual Speed – The ability to complete a low level cognitive task rapidly and accurately.

Visual Reaction Time – The ability to react quickly to a visual stimulus.

Visual Attention – The ability to focus or act upon relevant visual stimuli and to ignore irrelevant visual stimuli.

Short Term Visual Memory – The ability to retain visual memory for a short time before accurately acting on it.

Spatial Visual Counting

Substantial research shows that subitizing speed predicts counting speed. This is consistent with the concept that subitizing and counting are separate but overlapping processes and that counting builds on subitizing. Counting speed predicts adding speed and also predicts addition accuracy. Present a domino with 3 dots and most children will know instantly the correct number. They are using subitizing. Present a domino with 8 dots to a group and some "just know" the correct number. They are using subitizing plus counting. They recognize the number pattern as a composite of parts and as a whole. They "see" the domino as composed of two groups of 4 dots and as "one 8". This is an example of spatial counting and uses the technique of group and adds. Others will just use the slower and less efficient method of counting one item at a time.

The Spatial Visual Counting task presents an array of geometric figures that the patient is required to enumerate as quickly as possible. As in Flash the patient answers the question, "How many are there?" The patient is urged to answer as quickly as possible since response time is of vital importance. The array remains on the screen until the patient answers or designated time elapses.

Perceptual Analysis of Spatial Visual Counting

Subitizing – The ability to instantly know the number of items in a small set of elements without using any mathematical process.

Visual Spatial Working Memory – A spatial sketch-pad specialized for visual and/or spatial operations and a central executive responsible for coordinating and sequencing the activity of the spatial system.

Simultaneous Processing – The ability to integrate separate elements into a whole.

Visual Perceptual Speed – the ability to complete a low level cognitive task rapidly and accurately.

Short Term Visual Memory – The ability to retain visual material for a short time before accurately acting on it.

Visual Attention – The ability to focus or act upon relevant visual stimuli and to ignore irrelevant visual stimuli.

Temporal Visual Counting

This task requires counting a number of items presented in rapid succession. A specified target picture is displayed for a short interval. Then a number of the specified items are displayed for a short time interval. The patient is required to count the number items displayed. Dyscalculia may be affected by impairment in temporal visual processing. The concept of number, according to Spelke and Dehaene,[44] including the mental representation of quantities, may rely heavily on integration of sequential processing into a more abstract form, and thus require adequate temporal coding of numerical information. The task requires the ability to count rapidly, short term memory, and processing speed.

Perceptual Analysis of Temporal Counting

Temporal Visual Processing – The ability to process brief components of visual information presented rapidly.

Phonological Working Memory – The ability to process information in the dual task situation of counting the targets while retaining the total count in memory.

Sequential Processing – The ability to process information arriving in the brain in a serial order. The stimuli are temporal in nature and are not surveyable at any one time so the information is processed in a linear step-by-step fashion.

Visual Attention – The ability to focus or act upon visual stimuli and to ignore irrelevant stimuli.

Visual Reaction Time – the ability to react quickly to a visual stimulus.

Behavioral Optometry and Mathematics Learning Disability

Arithmetic deficits have not been in the forefront of behavioral optometric thinking. Very few mentions of dyscalculia, or even less significant problems, are found in our literature or heard from our lecture platforms, or even within our public information materials. This may be because good techniques for therapy are not available, or perhaps we have not paid enough attention to diagnosing visually related math problems. This is unfortunate because the number of individuals with math difficulties rivals reading problems.

Optometric interest in learning disabilities in mathematics has been mainly limited to deficiencies in various visual perception abilities.[45] Flax[46] discusses the relationship of visual factors to mathematics and points out that children who are unable to visualize spatially may have difficulty acquiring fundamental understanding of the relationship between numbers and quantity. He says that the ability to utilize spatial thinking, visualize and mentally manipulate shapes is important to full understanding of such subjects as trigonometry and geometry. Research and clinical experience has shown the importance of spatial relations[47] and simultaneous processing[48] to math. Optometric vision therapy to improve spatial relations and visualization is recommended. Computer programs, manipulatives, workbooks and other procedures[49] are readily available for this purpose.

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